Portfolio management | November 2026

Workshop overview

Efficient frontier workshop · the portfolio management case step by step

1. The case in brief

- **33 proposed portfolios**, made up of six assets: S&P 500, STOXX Europe 600, emerging markets, gold, US Treasuries 7-10 years and US real estate.
- **Daily data from 8 May 2006 to 7 May 2026**: annualised expected returns and risks, correlations measured over 5, 10 and 20 years.
- **Risk-free rate of 2.80%** a year: the average US 10-year rate over the 20 years of data in the case.
- Expected returns and risks are measured in each asset's currency: in dollars, and in euros for the STOXX Europe 600.

2. Learning objectives

Overall objective. By the end of the workshop, you will be able to select a portfolio and set the share to invest in it to reach a given expected return, risk or horizon, using the Sharpe ratio and the risk-free rate. You will also be able to assess how fragile the estimates are and what real risks an investment carries.

You will be able to:

1. explain why the risk of a portfolio is not the weighted average of the risks of its assets (the role of correlation);
2. read a risk and expected return chart, compare portfolios and spot those that are dominated;
3. calculate and interpret the Sharpe ratio, then identify, among the 33 proposed portfolios, the one with the best Sharpe ratio;
4. split your investment between this portfolio and the risk-free rate to target an exact expected return or risk, with or without borrowing (Tobin's separation theorem);
5. translate your risk aversion and investment horizon into a choice of portfolio and allocation;
6. assess the conditions and risks of borrowing to invest more;
7. measure how much the estimates depend on the measurement period and draw conclusions from this;
8. identify, beyond the estimates, the real-world factors that can weigh on a portfolio's performance.

3. How the workshop works

You work through the workshop in order, in 11 steps. At each step, you read the case question, explore the chart and its settings, read the boxes that help you answer, then write and save your answer under each sub-question. You then compare it with the solutions in step I, which explain each answer with the figures of the case. For better readability, switch to full screen with the “Full screen” button at the top right: charts and tables are shown larger.

Step	What you do	Questions
A · Understand	You vary the correlation between the S&P 500 and US Treasuries 7-10 years, then add the STOXX Europe 600 and gold to see how the efficient frontier shifts.	Preparation
B · Aim for 8%	You read the chart and the table of the 33 proposed portfolios. You move along the line that links the risk-free rate to a portfolio.	1a
C · The best	You compare the Sharpe ratios of the 33 proposed portfolios and the lines drawn from the risk-free rate.	1b, 1c, 1d
D · The mix	You set a risk or an expected return and read the share to invest. You take the risk aversion test.	2 and 3
E · Horizon	You set an amount you will need in 5 or 20 years and see which allocations, according to the model, cover it with the chosen level of certainty.	4 and 5
F · Aim higher	You work out how to reach a 12.39% expected return and under what conditions.	6
G · Period	You change the measurement period (5, 10 or 20 years) and observe what changes.	7
H · Stress test	You replay the actual history of 2006-2026 and its crises, then take on 11 challenges, each of which reveals a key point.	8
I · Solutions	You compare your answers with the detailed solutions, sub-question by sub-question.	All
J · Simulator	You build your own portfolio with the six assets: period, risk-free rate, allocation, leverage.	Used on the 3rd day of the course
K · Crises	You replay the simulator portfolio through real crises, against a 100% investment in US equities.	Going further

What is expected at the end

- A written answer to each of the **16 sub-questions** of the 8 case questions, backed by the figures read or calculated in the workshop.
- For question 8, **at least three justified key points**: the challenges in step H help you discover eleven.
- Your answers stay saved in your browser. You can export them as a PDF for yourself: you do not need to send them.
- The simulator (step J) will be used on the 3rd day of the course. You can already explore it if you wish. Steps A to I prepare you to use it well. Step K lets you go further: it shows how your portfolio behaves in crises.

4. The case questions

Question 1 Which portfolio should you choose if you want an expected return of exactly 8%?

1a. Using the chart and the table, which portfolio do you choose? Why?

1b. Which portfolio has the best Sharpe ratio? Compare it with the portfolios with an 8% expected return.

1c. By investing part of your money at the risk-free rate, how can you obtain an 8% expected return with this portfolio? What risk do you take, compared with the portfolios with an 8% expected return?

1d. Why is it the only risky portfolio you should hold, whatever your profile?

Question 2 Which portfolio should you choose if you want a risk of exactly 8%?

2a. Which portfolio should you choose if you want a risk of exactly 8%?

2b. What expected return do you obtain this way? Why is it the best choice for exactly 8% risk?

Question 3 What is your risk aversion based on the test?

3a. Take the risk aversion test. What annual risk do you accept?

3b. What share do you invest in the portfolio from question 1b and what share at the risk-free rate (or borrowed)?

Question 4 Which portfolio would you choose with a 5-year investment horizon, taking your risk aversion into account? Why?

Question 5 Which portfolio would you choose with a 20-year investment horizon, taking your risk aversion into account? Why?

Question 6 Which portfolio should you choose if you want an expected return of 12.39%?

6a. Among the 33 portfolios, which one reaches this expected return? With what risk?

6b. How can you reach it with the portfolio from question 1b? Under what condition and with what risks?

Question 7

7a. Estimate the expected returns and risks over the last 5, 10 and 20 years. How do they vary?

7b. Does the best Sharpe ratio always go to the same portfolio?

7c. What does this difference tell you?

Question 8 Beyond expected returns, which real-world factors could affect the performance of your portfolios? Identify at least three key points and justify them.

5. Key concepts

Concept	Key takeaway
Diversification and correlation	The risk of a mix is lower than the weighted average of the risks as soon as the correlation is below 1. The lower it is, the more diversification reduces risk.
Expected return E[r] and risk	E[r]: annualised average of past returns, used as an estimate. Risk: volatility, that is, the annualised standard deviation of returns.
Efficient frontier	The portfolios that offer the best expected return for each level of risk (Markowitz, 1952).
Risk-free rate and the line	Any mix of a portfolio with an investment at the risk-free rate lies on the line that links them. Beyond 100% in the risky portfolio, you borrow.
Sharpe ratio	(E[r] − risk-free rate) / risk: the expected return earned above the risk-free rate for each point of risk. It is the slope of the line that links the risk-free rate to the portfolio.
Separation theorem	With a risk-free rate, building a portfolio comes down to two decisions: which risky portfolio to hold, then what share to invest in it (Tobin, 1958).
Tangency portfolio (optimal risky portfolio)	The portfolio with the best possible Sharpe ratio when the six assets are mixed freely, without short selling. It is not one of the 33 proposed portfolios.
Risk aversion and horizon	Risk aversion sets the risk you accept. The investment horizon affects your capacity to bear that risk.
Borrowing and Lombard loans	Investing more than 100% of your stake in the risky portfolio means borrowing. In practice, this is a Lombard loan: the bank lends while holding the portfolio as collateral.
Fragility of the estimates	Expected returns are estimated from the past and depend on the measurement period. The data period is not the investment horizon.
Real-world risks	Expected returns do not tell the whole story: replaying the 2006-2026 crises reveals other factors that weigh on a real investment.

The workshop relies on past data for teaching purposes: it is neither a forecast nor investment advice.

Appendix: the case tables

Annualised expected returns and risks over 2006-2026 (daily data from 8 May 2006 to 7 May 2026), except for the horizon analysis. Source: case workbook, version of 6 November 2026.

The 33 proposed portfolios

No.	Expected return	Risk	S&P 500	STOXX Europe 600	Emerging markets	Gold	US Treasuries 7-10 years	US real estate	Sharpe ratio
1	3.66%	6.87%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%	0.13
2	4.57%	5.89%	2.14%	4.84%	2.28%	5.16%	84.87%	0.70%	0.30
3	5.06%	5.95%	3.01%	6.33%	1.37%	11.12%	78.17%	0.00%	0.38
4	6.00%	6.77%	0.80%	8.91%	8.22%	8.95%	60.68%	12.44%	0.47
5	7.38%	18.34%	0.00%	100.00%	0.00%	0.00%	0.00%	0.00%	0.25
6	7.50%	12.36%	1.41%	9.75%	9.28%	5.01%	39.09%	35.46%	0.38
7	7.68%	9.64%	11.68%	25.48%	9.31%	12.80%	31.33%	9.40%	0.51
8	7.96%	19.36%	0.00%	0.00%	100.00%	0.00%	0.00%	0.00%	0.27
9	8.00%	9.55%	11.13%	16.79%	16.23%	18.62%	28.58%	8.65%	0.54
10	8.00%	14.30%	6.84%	12.75%	9.24%	0.92%	31.65%	38.61%	0.36
11	8.12%	13.66%	9.12%	21.87%	27.05%	1.27%	18.80%	21.89%	0.39
12	8.13%	8.43%	14.89%	5.12%	10.64%	28.21%	36.01%	5.14%	0.63
13	8.26%	10.31%	0.69%	8.29%	13.53%	28.32%	28.53%	20.64%	0.53
14	8.32%	11.95%	22.04%	10.91%	27.71%	3.65%	23.89%	11.79%	0.46
15	8.48%	11.60%	18.10%	17.83%	23.21%	11.10%	19.68%	10.09%	0.49
16	8.49%	11.53%	14.40%	13.68%	5.10%	15.03%	28.40%	23.38%	0.49
17	8.61%	9.89%	12.26%	3.83%	19.41%	28.07%	25.92%	10.52%	0.59
18	8.62%	14.98%	11.52%	2.27%	11.56%	5.38%	28.41%	40.85%	0.39
19	8.66%	11.73%	16.00%	6.52%	23.08%	13.67%	22.45%	18.28%	0.50
20	8.68%	15.08%	6.02%	33.22%	19.40%	6.95%	8.24%	26.16%	0.39
21	8.79%	13.17%	6.98%	43.34%	11.62%	20.67%	5.86%	11.53%	0.46
22	8.93%	12.10%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	0.51
23	9.12%	12.20%	5.30%	7.97%	21.78%	28.24%	14.52%	22.19%	0.52
24	9.35%	11.32%	6.02%	9.55%	16.15%	43.31%	13.74%	11.23%	0.58
25	9.46%	11.94%	12.87%	23.96%	15.05%	34.10%	6.54%	7.48%	0.56
26	9.52%	13.99%	11.14%	16.54%	13.91%	21.98%	8.79%	27.64%	0.48
27	9.65%	14.38%	6.00%	8.80%	7.36%	28.85%	12.78%	36.21%	0.48
28	9.75%	12.47%	38.03%	4.63%	0.94%	18.22%	22.06%	16.11%	0.56
29	10.00%	22.05%	6.13%	13.48%	3.29%	4.98%	6.20%	65.91%	0.33
30	10.00%	14.90%	23.29%	14.73%	15.38%	17.37%	4.76%	24.47%	0.48
31	10.92%	29.94%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.27
32	11.27%	17.87%	0.00%	0.00%	0.00%	100.00%	0.00%	0.00%	0.47
33	12.39%	19.52%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.49

Rows in yellow are the single-asset portfolios. Sharpe ratio = (expected return − risk-free rate of 2.80%) / risk.

The six assets and the risk-free rate

Asset	Expected return	Risk
S&P 500 (USD)	12.39%	19.52%
STOXX Europe 600 (EUR)	7.38%	18.34%
Emerging markets (USD)	7.96%	19.36%
Gold (USD)	11.27%	17.87%
US Treasuries 7-10 years (USD)	3.66%	6.87%
US real estate (USD)	10.92%	29.94%
Risk-free rate	2.80%	

Correlations of daily returns (2006-2026)

	S&P 500	STOXX Europe 600	Emerging markets	Gold	US Treasuries 7-10 years	US real estate
S&P 500	1.00	0.60	0.44	0.05	−0.30	0.76
STOXX Europe 600	0.60	1.00	0.65	0.04	−0.25	0.40
Emerging markets	0.44	0.65	1.00	0.16	−0.18	0.30
Gold	0.05	0.04	0.16	1.00	0.21	0.05
US Treasuries 7-10 years	−0.30	−0.25	−0.18	0.21	1.00	−0.16
US real estate	0.76	0.40	0.30	0.05	−0.16	1.00

Expected returns and risks by data period

Asset	Return 5 years	Risk 5 years	Return 10 years	Risk 10 years	Return 20 years	Risk 20 years
S&P 500 (USD)	14.03%	16.91%	16.05%	18.05%	12.39%	19.52%
STOXX Europe 600 (EUR)	10.27%	14.13%	9.99%	15.46%	7.38%	18.34%
Emerging markets (USD)	9.31%	16.61%	11.74%	16.32%	7.96%	19.36%
Gold (USD)	20.54%	17.60%	14.16%	15.75%	11.27%	17.87%
US Treasuries 7-10 years (USD)	−0.70%	7.68%	0.95%	6.62%	3.66%	6.87%
US real estate (USD)	6.20%	18.77%	8.22%	20.72%	10.92%	29.94%

The 5, 10 and 20-year periods are data periods, not investment horizons.